

## THE KAVERY ENGINEERING COLLEGE

(An Autonomous Institution)

M.E DEGREE END SEMESTER THEORY EXAMINATIONS, NOV /DEC 2024

First Semester

Applied Electronics

PAMAET2411 – Applied Mathematics for Electronics Engineering

(Statistical Tables can be Permitted)

Regulations - 2024

Duration : 3 Hrs

Maximum : 100 Marks

## PART – A (ANSWER ALL QUESTIONS) (Marks 10\*2=20)

| Q.No | Questions                                                                                                                                   | BLT | CO | Marks |
|------|---------------------------------------------------------------------------------------------------------------------------------------------|-----|----|-------|
| 1.   | Define tautology.                                                                                                                           | RE  | 1  | 2     |
| 2.   | What are the types of fuzzy proposition?                                                                                                    | RE  | 1  | 2     |
| 3.   | For the following density function $f(x) = a e^{- x }$ , $-\infty < x < \infty$ , find the value of "a".                                    | UN  | 2  | 2     |
| 4.   | Six coins are tossed 6400 times. Using the poisson distribution, what is the approximate probability of getting six heads 10 times.         | UN  | 2  | 2     |
| 5.   | Find the marginal density function of X and Y if $f(x,y) = \frac{2}{5}(2x + 5y)$ , $0 \leq x \leq 1, 0 \leq y \leq 1$ .                     | UN  | 3  | 2     |
| 6.   | The two lines of regression are $3x + 2y = 26$ , $6x + y = 31$ . Find the mean value of x and y.                                            | UN  | 3  | 2     |
| 7.   | Verify whether the sine wave process $\{X(t)\} = Y \cos \omega t$ , where Y is uniformly distributed in (0,1) is a strict sense stationary. | UN  | 4  | 2     |
| 8.   | State any two properties of poisson process.                                                                                                | RE  | 4  | 2     |
| 9.   | Define Kendall's notation.                                                                                                                  | RE  | 5  | 2     |
| 10.  | If $\lambda = 6$ and $\mu = 8$ , find the probability of at least 10 customers in the system by considering an M/M/1 queueing system.       | UN  | 5  | 2     |

## PART – B (ANSWER ALL QUESTIONS) (Marks 5\*13 = 65)

| Q.No | Questions                                                                                                                                                    | BLT | CO | Marks |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|----|-------|
| 11.  | a i Explain the classification of fuzzy proposition.                                                                                                         | UN  | 1  | 6     |
|      | ii For the inference rule $[(A \rightarrow B) \cap (B \rightarrow C)] \rightarrow [A \rightarrow C]$ show that the rule is a quasi tautology for fuzzy sets. | UN  | 1  | 7     |
|      | Or                                                                                                                                                           |     |    |       |
|      | b i What type of operators are used in fuzzy expression?                                                                                                     | UN  | 1  | 6     |
|      | ii Explain the different types of fuzzy quantifier with examples.                                                                                            | UN  | 1  | 7     |

12. a i Three identical boxes contain red and white balls. The first box contains 3 red and 2 white balls, the second box has 4 red and 5 white balls, and the third box has 2 red and 4 white balls. A box is chosen very randomly and a ball is drawn from it. If the ball that is drawn out is red, what will be the probability that the second box is chosen? AP 2 6
- ii Out of 800 families with 4 children each, how many families would be expected to have i) 2 boys and 2 girls ii) At least 1 boy iii) at most 2 girls and iv) children of both genders. Assume equal probabilities for boys and girls. AP 2 7
- Or
- b i In a certain factory producing razor blades, there is a small chance  $\frac{1}{500}$  for any blade to be defective. The blades are supplied in packets of 10. Use poisson distribution to calculate the approximate number of packets containing i) no defective blade ii) at least 1 defective blade iii) at most 1 defective blade in a consignment of 10,000 packets. AP 2 6
- ii The mileage which car owners get with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40,000km. Find the probabilities that one of these tyres will last i) at least 20,000 km and ii) at most 30,000km. AP 2 7
13. a i The joint probability mass function of (X,Y) is given by  $P(X,Y) = K(2X + 3Y)$ ,  $X = 0,1,2$  and  $Y = 1,2,3$ . Find the all marginal and conditional probability distributions. AP 3 7
- ii The joint probability density function of (x, y) is given by  $f(x,y) = kxye^{-(x^2+y^2)}$ ,  $x, y > 0$ . Show that x & y are independent. 3 6
- Or
- b The joint probability density function of two dimensional random variable (x, y) is given by  $f(x, y) = x + y$ ,  $0 < x, y < 1$ . Find the correlation co efficient between x and y. AP 3 13
14. a The process  $\{X(t)\}$  whose probability distribution under certain conditions is given by  $P(X(t) = n) = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,3, \dots \\ \frac{at}{1+at}, & n = 0 \end{cases}$ . Show that it is not stationary. AP 4 13
- Or
- b The transition probability matrix of a Markov chain  $\{X_n\}$ , three states 1, 2 and 3 is  $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$  and the initial distribution is  $P^{(0)} = (0.7, 0.2, 0.1)$ . Find (i)  $P\{X_2 = 3\}$  and (ii)  $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$ . AP 4 13

15. a Customers arrive at a one-man barber shop according to a poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. The service time is exponentially distributed. If an hour is used as a unit of time, then
- What is the probability that a customer need not wait for a hair cut?
  - What is the expected number of customer in the barber shop and in the queue?
  - How much time can a customer expect to spend in the barber shop?
  - Find the average time that a customer spend in the queue.
  - Estimate the fraction of the day that the customer will be idle.

What is the probability that there will be 6 or more customers?

Or

- |     |                                                                                                                                                                                            |    |   |   |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|---|---|
| b i | Derive Pollaczek- Khintchine formula.                                                                                                                                                      | AP | 5 | 6 |
| ii  | A repair facility shared by a large number of machine has 2 series stations with respective service rates of 2 per hour and 3 per hour. If the average rate of arrival is 1 per hour, find | AP | 5 | 7 |
|     | i)The average number of machines in the systems.                                                                                                                                           |    |   |   |
|     | ii)The average waiting time in the system.                                                                                                                                                 |    |   |   |
|     | iii)Probability that both service stations are idle.                                                                                                                                       |    |   |   |

*PART - C (Marks 1\*15 = 15)*

| <i>Q.No</i> | <i>Questions</i>                                                                                                                                                                                                                                                                                      | <i>BLT</i> | <i>CO</i> | <i>Marks</i> |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|-----------|--------------|
| 16. a i     | In a bolt factory machines A,B,C manufacture respectively 25%, 35% and 40% of the total. In their output. 5%, 4% and 2% are defective bolts. A bolt is drawn at a random from the production and is found to be defective. What are the probabilities that it was manufactured by machines A,B and C. | AP         | 2         | 7            |
| ii          | Consider the random process $X(t) = A \sin(\omega t + \Phi)$ , where A and $\omega$ are constant, $\Phi$ is a random variable uniformly distributed in $(0, 2\pi)$ . Find the auto correlation function of the process.                                                                               | AP         | 2         | 8            |
| Or          |                                                                                                                                                                                                                                                                                                       |            |           |              |
| b           | If "X" & "Y" are two independent random variables follows exponential distribution with parameter 1, then find the probability density function of $U = \frac{X+Y}{2}$ .                                                                                                                              | AP         | 3         | 15           |